ESTIMATE OF THERMOELECTRIC EFFECTS IN THE ELECTRODE REGION OF A GAS DISCHARGE

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In calculating thermal fluxes in a plasma, in certain studies [1-3] account is taken for the thermoelectric effects. A one-dimensional model problem on the heating on a medium as a result of Joule dissipation was examined in [1], where it was shown that the effects associated with energy transport by electrons lead to asymmetric temperature distribution between the electrodes. The problem of the linear pinch discharge in hydrogen was solved in [2]. One of the results of the present study will be to establish the asymmetry of the thermal fluxes to the anode and cathode. An attempt to estimate the potential drop in the region near the electrode in application to the cathode spot was made in [3]. In this case the problem will be three-dimensional and the thermoelectric effects should be amplified as a result of the large temperature gradients. The solution of [3] for the limiting case of strong currents, obtained with the aid of numerical methods, does not permit finding the parameter distribution in the region near the electrode or analyzing the influence of various effects.

In the following we use the scheme adopted in [3] to examine the influence of theromelectric effects on the thermal flux and potential fall in the region near the electrode for a broad range of parameters.

It is well known that current flow of sufficiently high strength in a plasma is accompanied by the formation of spots on the electrodes. We shall examine the region near the elecrode at a distance from the spot on the order of the spot size. Then all the quantities in this region can be considered to depend only on the radius. This is possible in the case of a single isolated spot, when in its vicinity the normal component of the current density on the surface of the electrode is zero and the current in the plasma above the electrode spreads diffusely, without constricting.

In this case the processes which are defined by the emission properties of the electrodes and which take place at a distance on the order of the electron mean free path from the electrode surface are not examined. We specify the boundary conditions on a sphere of radius r_0, terming it arbitrarily an electrode, and by virtue of symmetry we can restrict ourselves to a hemisphere.

We write the current, heat flux, and energy equations

$$j_r = \sigma E_r + \Phi \frac{dT}{dr}, \quad q_r = -\lambda \frac{dT}{dr} + \psi j_r, \quad \frac{1}{r^2}\frac{d}{dr}(r^2 q_r) = j_r E_r \tag{1}$$

Here j_r, q_r are the current density and thermal flux, E_r is electric field intensity, T is temperature, r is the coordinate. The transport coefficients σ, Φ, λ, ψ are taken for fully ionized hydrogen from [2]

$$\begin{gathered}\sigma = \alpha T^{3/2}, \quad \Phi = \alpha\gamma T^{3/2}, \quad \lambda = kT^{5/2}, \quad \psi = -\zeta T \\ \alpha = 1.1\cdot 10^{-3}, \quad \gamma = 0.69\cdot 10^{-4}, \quad k = 8.7\cdot 10^{-12}, \quad \zeta = 2.85\cdot 10^{-4}\end{gathered} \tag{2}$$

(The SI unit system is used.)

We note that the coefficient ζ combines two different energy transport mechanisms: enthalpy transport by electron flux and the thermoeffect [4], where the basic contribution is that of electron enthalpy (78%).

We examine the steady-state process with constant pressure (it is not difficult to see that in this case the concentration gradient is balanced by the temperature gradient and the system of equations (1) is closed since the concentration does not appear in this system), therefore in the current equation there is no term with electron pressure gradient. Introducing the variables

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$$p = (T/T_\infty)^{5/2}, \qquad n = k_1^{1/2} I / r T_\infty^{5/2}$$

proposed by Smay [3], we obtain from (1) and (2)

$$-p\frac{d^2p}{dn^2} = \frac{2}{5}\left(\frac{dp}{dn}\right)^2 - 2\frac{k_2}{k_1^{1/2}}\frac{dp}{dn} + \frac{20}{7}, \quad k_1 = {}^7/_{32}\,\pi^2\alpha k, \quad k_2 = (\zeta - \gamma)/4k\pi, \quad I = 2\pi r^2 j_r \qquad (3)$$

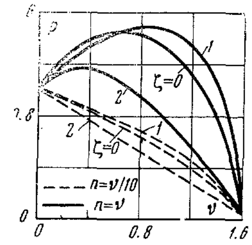

Fig. 1

Here I is the given total current through the hemisphere, T_∞ is the temperature at an infinitely distant point. The temperature at the electrode and at infinity are given as the boundary conditions

$$n = n_e,\ p = p_e, \qquad n = 0, \quad p = 1 \qquad (4)$$

We obtain the following expressions for the total thermal flux and the potential relative to the infinitely distant point

$$Q = -\zeta T_\infty I p^{2/5}\left[1 - \left(\frac{4\pi}{5}\frac{kk_1^{1/2}}{\zeta}\right)\frac{dp}{dn}\right], \quad V = \zeta T_\infty\left\{p^{2/5}\left[1 - \left(\frac{4\pi}{5}\frac{kk_1^{1/2}}{\zeta}\right)\frac{dp}{dn}\right] + \left(\frac{4\pi}{5}\frac{kk_1^{1/2}}{\zeta}\right)\frac{dp}{dn}\bigg|_\infty - 1\right\} \qquad (5)$$

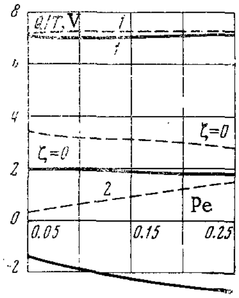

Fig. 2

The direction of the normal is taken outward, so that $Q < 0$ if the thermal flux is directed to the electrode. In the anode case $I > 0$, in the cathode case $I < 0$.

Equation (3) together with the condition (4) was integrated numerically on a digital computer using the Runge-Kutta method. The solution changes qualitatively for sufficiently large value of the parameter n_e (region of "strong" currents). A maximum appears in the temperature distribution curve, therefore as an example we consider two cases: "small" ($n_e = 0.16$) and "large" ($n_e = 1.6$) currents, which correspond to current densities 10^6 and 10^7 A/m^2 for $T_\infty = 20{,}000°$ K.

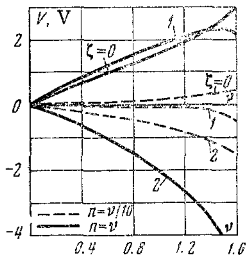

Fig. 3

1. Small Currents. (The dashed curves in the figures correspond to this case.) Figure 1 shows curves of the function p(n), characterizing the temperature distribution. The solution shows that as a result of the thermoeffect and enthalpy transport by the electrons the temperature in the anode region (curve 1) increases, while in the cathode region (curve 2) it decreases in comparison with the case of energy transport by thermal conduction only (curve $\zeta = 0$). There is a corresponding change of the temperature gradients at the electrode.

Figure 2 shows the reduced thermal fluxes Q/I as a function of the electrode temperature parameter p_e. The weak influence of the electrode temperature on the thermal fluxes is explained by the large difference between the plasma and electrode temperatures. The thermal flux to the anode (curve 1) is determined by the sum of the thermoeffect and thermal conduction, while the thermal flux to the cathode (curve 2) is determined by their difference. For $\zeta = 0$ (only thermal conduction) the thermal fluxes to the cathode and anode are the same.

We note that the thermoelectric effects and enthalpy transport by electrons show up in the form of an increase of the temperature gradient and a corresponding growth of the thermal conductivity. The second term in the expression (1) for the thermal flux is small because of the low electrode temperature.

Another interesting case, in which the thermal flux contains a term proportional to the current, will be the discharge in a partly ionized gas, where in the cold layer near the electrode the electron temperature is considerably higher than the gas temperature and the principal contribution to the thermal flux is provided by the term proportional to the current. Such a scheme was examined in [5]. The calculations showed that in this case as well there is considerable asymmetry of the thermal fluxes to the electrode.

The potential distribution is shown in Fig. 3. The negative potential near the anode (curve 1) is formed as a result of compensation of the large temperature gradient required to provide the given current [thermogenerator regime, see the first equation (1)].

2. Large Currents. (This case is shown in the figures by continuous curves.) As indicated in [3], for large currents the temperature distribution curve has a maximum (Fig. 1). The maximum is due to Joule heat release (curve $\zeta = 0$) and is shifted toward the anode (curve 1) as a reslut of the thermoeffect and enthalpy transport by electrons. Just as in the case of the "small" currents, the temperature in the cathode region (curve 2) is lower than in the anode region.

With ten-fold increase of the current the reduced thermal currents of Fig. 2 change only slightly. Only in the cathode case (curve 2) does the thermal flux change direction (heat travels from the electrode into the plasma).

The potential distribution is shown in Fig. 3. The anode potential (curve 1) is positive, but begins to decrease near the electrode for the reason noted above. As a result the potential distribution has a peak near the anode. A similar result was obtained in [1], where the electric field intensity was calculated and it was shown that E vanishes near the anode.

The other curves (curve 2 and $\xi = 0$) do not differ qualitatively from the dashed curves in Fig. 3.

Comparison of these results with the experimental data is difficult because of the near-electrode effects associated with electrode emission properties, [6] which are of the same order. Electron emission from the metal, except for autoelectronic emission, requires the expenditure of an energy of about 4 eV (work function), which is then released at the anode. This must lead to still greater asymmetry of the thermal fluxes. The potential jumps which arise near the electrodes at a distance on the order of the electron mean free path will be the cause of additional energy supply to the electrodes. This additional contribution may be different for the cathode and anode.

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